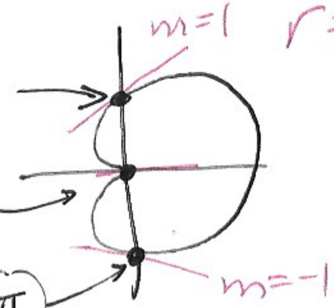
ND SLOPE OF TANGENT LINE AT y-INT OF $r = 1+\cos\theta$

$\underbrace{\text{SLOPE OF TANGENT LINE}}_{\frac{dy}{dx}}$

$r = f(\theta)$



$r = 0 = 1+\cos\theta$

$\cos\theta = -1$

$\theta = \pi$

$$\left.\begin{aligned} x &= r\cos\theta = f(\theta)\cos\theta \\ y &= r\sin\theta = f(\theta)\sin\theta \end{aligned}\right\}$$

PARAMETRIC EQUATIONS IN PARAMETER θ

PRODUCT, NOT $\frac{d}{d\theta}\sin\theta$

$$\frac{dy}{dx} = \underbrace{\frac{\frac{d}{d\theta} f(\theta)\sin\theta}{\frac{d}{d\theta} f(\theta)\cos\theta}}_{\text{FROM } \frac{dy}{dx} \text{ IN PARAMETRIC}} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

$$= \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$$

$x = (1+\cos\theta)\cos\theta$

$y = (1+\cos\theta)\sin\theta$

$x = (1+\cos\theta)\cos\theta$

$y = (1+\cos\theta)\sin\theta$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-\sin^2\theta + (1+\cos\theta)\cos\theta}{-\sin\theta\cos\theta - (1+\cos\theta)\sin\theta}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = \frac{-1^2 + \cancel{(1+0)0}}{\cancel{-1(0)} - (1+0)\cdot 1} = 1$$

$\cos\frac{\pi}{2} = 0$

$\sin\frac{\pi}{2} = 1$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{3\pi}{2}} = \frac{-(-1)^2 + \cancel{(1+0)0}}{\cancel{-(-1)0} - (1+0)(-1)} = -1$$

$\cos\frac{3\pi}{2} = 0$

$\sin\frac{3\pi}{2} = -1$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi} = \frac{\cancel{-0^2} + \cancel{(1+-1)(-1)}}{\cancel{-0(-1)} - \cancel{(1+-1)0}} = \frac{0}{0}$$

DOESN'T NECESSARILY MEAN $\frac{dy}{dx}$ DNE

$\cos\pi = -1$

$\sin\pi = 0$

$$\lim_{\theta\to\pi}\frac{dy}{dx} = \lim_{\theta\to\pi}\frac{-\sin^2\theta + \cos\theta + \cos^2\theta}{-\sin\theta\cos\theta - \sin\theta - \sin\theta\cos\theta}$$

$$= \lim_{\theta\to\pi}\frac{\cos 2\theta + \cos\theta}{-\sin 2\theta - \sin\theta} \qquad \frac{0}{0}$$

$$\text{L'H} \quad = \lim_{\theta\to\pi}\frac{-2\sin 2\theta - \sin\theta}{-2\cos 2\theta - \cos\theta} \qquad \frac{0}{-2(1)-(-1)} = \frac{0}{-1} = 0$$

$$= 0$$

PE OF T.L. TO $r=f(\theta)$ IS

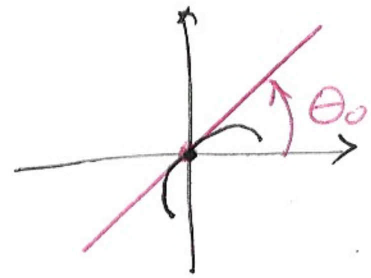
$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

OR

$$x = f(\theta)\cos\theta$$
$$y = f(\theta)\sin\theta$$
$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

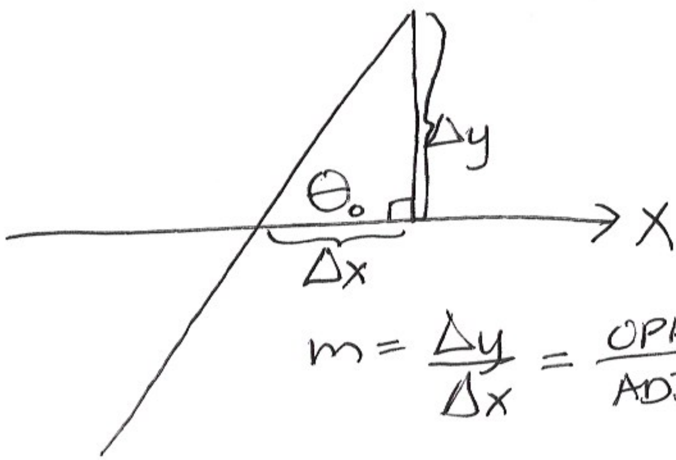
NOTE: IF $r=0$

WHEN $\theta=\theta_0$



$$\frac{dy}{dx} = \frac{\left.\frac{dr}{d\theta}\right|_{\theta=\theta_0}\sin\theta_0 + 0}{\left.\frac{dr}{d\theta}\right|_{\theta=\theta_0}\cos\theta - 0} = \tan\theta_0$$

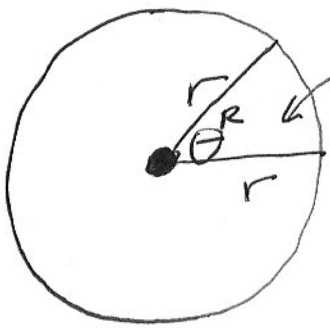
A LINE THAT MAKES AN ANGLE OF θ_0 WITH RIGHT SIDE OF X AXIS HAS SLOPE $\tan\theta_0$



$$m = \frac{\Delta y}{\Delta x} = \frac{OPP}{ADJ} = \tan\theta_0$$

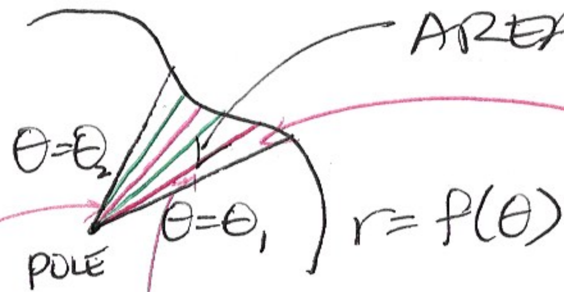
IE. THE LINE THROUGH THE POLE $\theta=\theta_0$ IS TANGENT TO THE CURVE @ THE POLE

4 AREA OF POLAR REGIONS



AREA OF SECTOR $= \frac{\theta^R}{2\cancel{\pi}}\cancel{\pi} r^2 = \frac{1}{2}r^2\theta^R$

AREA OF CIRCLE $= \pi r^2$



AREA OF REGION

AREA OF SUBREGION 1

$r_1 \approx$ CONSTANT IF $\Delta\theta \approx 0$

$A_1 \approx \frac{1}{2}r_1^2\Delta\theta$

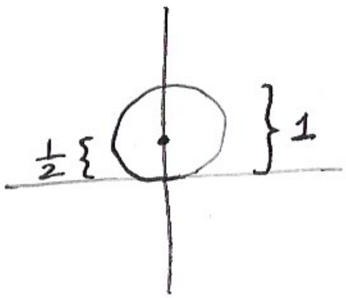
ANGLE $= \Delta\theta$

GLE
TWEEN
θ_1 AND $\theta=\theta_2$ DIVIDED INTO n SUB-ANGLES

AREA OF REGION $\approx \sum_{i=1}^{n} A_i = \sum_{i=1}^{n}\frac{1}{2}r_i^2\Delta\theta$

$$\text{AREA} = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{2}r_i^2\Delta\theta = \int_{\theta_1}^{\theta_2}\frac{1}{2}r^2\,d\theta$$

CALL $r = \sin\theta$



$A = \pi\left(\frac{1}{2}\right)^2$

$A = \frac{\pi}{4}$

$$\int_0^{\cancel{2\pi}\ \pi} \frac{1}{2} r^2\, d\theta$$

$$= \int_0^{\cancel{2\pi}\ \pi} \frac{1}{2}\sin^2\theta\, d\theta$$

$$= \frac{1}{2}\cdot\frac{1}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right)\Big|_0^{\cancel{2\pi}\ \pi}$$

$$= \frac{1}{4}\left((\cancel{2}\pi - 0) - \frac{1}{2}(0-0)\right)$$

$$= \cancel{\frac{\pi}{2}}\ \frac{\pi}{4}$$

IF θ GOES FROM 0 TO 2π,
THE CIRCLE IS DRAWN TWICE

$$\int \sin^2\theta\, d\theta$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$
$$2\sin^2\theta = 1 - \cos 2\theta$$
$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$= \frac{1}{2}\int (1 - \cos 2\theta)\, d\theta$$

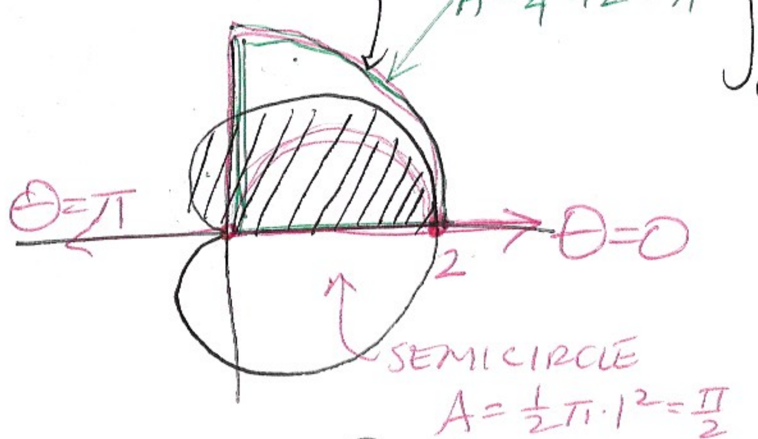
$$= \frac{1}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right) + C$$

$$\int \cos^2\theta\, d\theta = \frac{1}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right) + C$$

$$2\int_0^{\frac{\pi}{2}} \frac{1}{2}\sin^2\theta\, d\theta$$

IF YOU PROVE $r = \sin\theta$ IS SYM OVER $\theta = \frac{\pi}{2}$

$r = 1 + \cos\theta$



$$\int_0^{\pi} \frac{1}{2}(1+\cos\theta)^2\, d\theta$$

$$= \frac{1}{2}\int_0^{\pi} (1 + 2\cos\theta + \cos^2\theta)\, d\theta$$

$$= \frac{1}{2}\left(\theta + 2\sin 2\theta + \frac{1}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right)\right)\Big|_0^{\pi}$$

$$= \frac{1}{2}\left(\frac{3}{2}\theta + \frac{9}{4}\sin 2\theta\right)\Big|_0^{\pi}$$

$$= \frac{1}{2}\left(\frac{3}{2}(\pi - 0) + \cancel{\frac{9}{4}(0-0)}\right)$$

$$= \frac{3\pi}{4} > \frac{\pi}{2} \text{ SANITY CHECK}$$

$< \pi$ SANITY CHECK

SOLVE $r = 0$

$1 + \cos\theta = 0$

$\cos\theta = -1$

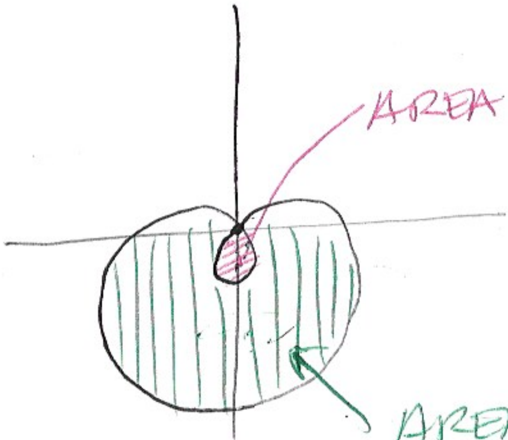
$\theta = \pi$

$\theta \in [0, 2\pi]$

$r = 1 - 2\sin\theta$

AREA INSIDE LIMAÇON =

$$\int \frac{1}{2}(1-2\sin\theta)^2\, d\theta$$



AREA INSIDE INNER LOOP

+

AREA INSIDE OUTER LOOP

AND OUTSIDE INNER LOOP